Design of a Reinforced Concrete Railroad Arch Bridge

O. T. Allen
C. S. Holcomb

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Design of a Reinforced Concrete
Railroad Arch Bridge.

A Thesis presented by

O.T.Allen and C.S.Holcomb

to the
President and Faculty
of the
Armour Institute of Technology
for the degree
of
Bachelor of Science in Civil Engineering
Having completed the prescribed course of study in
Civil Engineering.

Chicago--June--1906.

Howard M. Raymond
Dean of Engineering Studies.

Dean of Cultural Studies.

Alfred E. Phillips
Professor of Civil Engineerin

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The Design of a Reinforced Concrete Railroad Arch Bridge.

Data:-

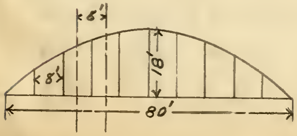
The data assumed for the design of this bridge was as follows:- The line of stress was taken as parabolic in form,fixed at the ends,of 80 foot span,and 18 foot rise. The abutments were assumed as resting upon pile foundations, the bases being 18 feet below the springing of the line of stress. The bridge was designed for a single track and the extreme width taken as 20 feet 6 inches.

Vertical Dead Load.

The weight of concrete was taken as 150# per cubic foot,and that of the earth filling as 100# per cubic foot. The factor of safety was taken as 2 for dead load.

With data obtained from the plans of several similar bridges an approximate section for this arch was drawn. The section was divided into 10--8 foot lengths or panels; the load considered as applied at these panel points consisted of all of the material within vertical planes 4 feet either side of the panel point. The weight of this material for each panel point was obtained by scaling the area in square feet between the two vertical planes,multiplying this by the thickness, taken as 1 foot,and then multiplying by the assumed weight of the material. The above is shown in figure #1.

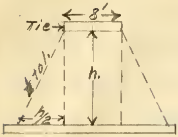
Figure #1



Vertical Live Loads.

The live load was assumed as 50,000# on axles 5 foot centers, which is equal to 10,000# per linear foot of track;this load was taken

Figure #2

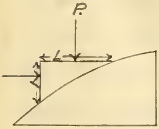


as distributed uniformly over ties 8 feet long. Owing to the nature of the earth filling the load would be distributed over an area which increases with the depth; the outer edge of this area was assumed as extending downward from the end of the tie at a slope of 1/2 to 1. The load on the area would vary from zero at the outer edge to the maximum under the tie. Let P be this maximum pressure; then as may be seen from figure #2 :- $Px8 + \frac{P}{2}x\frac{h}{2}x2 =$ 10,000# or $P = \frac{20,000\#}{h + 16}$. Adding 50% for impact $P = \frac{30,000\#}{h + 16}$ A more extended description of the above may be found in "The Calculation of the Stresses and the Practical Design of Structures of Steel Concrete" by Mr.Walter W.Colpitts.

Horizontal Dead Loads.

The horizontal dead loads were figured as follows:- Let P be the total vertical pressure acting over the area L,

Figure #3



q the total horizontal pressure acting over the area l; if the unit pressure on l equals that on L, then P:q = L:l or q = Pl/L but according to Rankines theory of earth pressure the horizontal pressure is 1/3 the vertical pressure when the angle of repose of earth is 30 degrees; therefore q = Pl/3L which was the formula used in figuring the horizontal loads. P was taken as the vertical panel load, L the panel length or 8 feet, and l the difference between the ordinates to the parabola at points 4 feet either side of the panel point.

Horizontal Live Loads.

These were calculated by the same formula as the horizontal dead loads except that P was now taken as a vertical live panel load.

Stresses- - - - - - - - - - - Bending Moments.

The vertical bending moments were figured from the tables given on page 70 in "Trusses and Arches-Part 3" by Prof. Chas. E. Greene. The formula is M = mcW, m being the constant taken from the tables, c the half span = 40 feet, and W the load at the panel point. The results obtained, together with the maximum and minimum values at each panel point, are shown in tables #3+5 The results shown in tables #3+5 are in inch pounds and therefore 12 times as large as the values given by Prof. Greenes tables which are calculated to give the bending moments in foot pounds. The horizontal bending moments were figured by the formula and table given on page 38 of Mr. Colpitt's book which was mentioned before. This formula is M = mkW where m is the constant taken from the table, k the rise of the arch or 18 feet, and W the horizontal panel load. This table gives the values at the different panel points directly in inch pounds. These results are shown in tables #4+6.

Thrusts.

The horizontal thrusts due to vertical loads were figured by means of the table on page 70 of Prof. Greene's book. The formula used was H = mcW/k where H is the horizontal thrust, m the constant of the table, c the half span in feet, k the rise in feet, and W the vertical panel load. The summation of the horizontal thrusts for the various panel

points gives the total horizontal thrust of the arch;this thrust was found for both the dead and live loads and the results are shown in table # / .

The thrust due to the horizontal loads is the sum of the horizontal loads at the various panel points. The results are shown on table # / .

Shear.

The values for shear due to vertical loads were obtained by means of Prof. Greene's shear table given on page 70 of his book; the formula is V = nW where V is the shear, n the constant of the table, and W the vertical load at the panel considered. It was considered that the shear due to horizontal loads was too small to affect the results,especially as the sections figured out as explained later for bending moment were very much larger than were required by the shear. No table of the shear for the horizontal loads was therefore figured.

Temperature.

Owing to the fact that any change in temperature causes an expansion or contraction in the arch, thrusts and bending moments will be set up which must be taken into account in figuring the sections. With a fall of temperature negative thrusts and positive bending moments will be produced, while with a rise of temperature positive thrusts and negative bending moments will be produced. Prof. Greene has derived on pages 76 and 77 in his book, formulae for finding the thrusts and bending moments under these conditions. These formulae are stated by Mr.Colpitts in his book on page 33 in the convenient forms:-

$H_t = \frac{45\ teEI}{4x\ 144k^2}$ $M_t = \frac{15\ teEI}{4\ x\ 12k}$ (at the crown) where:-

t is the extreme rise or fall in temperature.
e " " coefficient of expansion of concrete.
E " " modulus of elasticity of concrete.
I " " moment of inertia in inches of the section at the crown about the neutral axis.
H_t " " horizontal thrust in pounds due to change in temperature.
M_t " " bending moment in inch " " " " " "
k " " rise of the arch in feet.

Assuming $t = 75$ degrees "F", $e = .0000055$ per degree "F" $E = 3{,}000{,}000\#$, the formula for H_t reduces to $H_t = 96\ I/k^2$ and the formula for M_t to $M_t = 387\ I/k$.

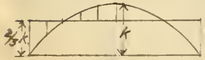
The value of I is obtained from the formula $I = Mx'/f_c$ where M is the bending moment at the section, x' the distance in inches from the compression face to the neutral axis, and f_c the intensity of the compressive stress in pounds per square inch on the extreme fibres of the concrete, and due to bending moment only. The method of obtaining f_c will be shown later.

If the temperature moment at the crown be multiplied by the following factors, the temperature moments at the various panel points will be obtained.

Panel points	Abutment	1	2	3	4	5
Factors	2.0	.92	.08	.52	.88	1.0

The temperature bending moment is obtained by multiplying the temperature thrust by the ordinate between the parabola and the equilibrium polygon for temperature bending moments. The factors given above are the ratios of the ordinates at the various panel points to the ordinate at the crown.

Figure #4.



For the reason stated under Shear due to horizontal loads, no temperature shears were worked out.

FORMULAS FOR MOMENT OF RESISTANCE IN REINFORCED BEAMS.

From "A Treatise on Concrete Plain and Reinforced" by Taylor and Thompson.

The theory given below is the straight line theory of the distribution of stress with the concrete taking no tension. The following assumptions are made:-

1-A plane section before bending remains plane after bending.
2-Tension is borne entirely by the steel.
3-Initial tension or compression is absent in the steel.
4-Adhesion of steel to concrete is perfect.
5-Modulus of elasticity of concrete within the usual limits of stress is a constant.

- -

NOTATION.

h = height of beam.
b = breadth of beam. (1')
p = ratio of cross section of steel to cross section of beam (above the center of gravity of steel)
C = unit pressure on outside fibre of concrete. (1100#□")
S = unit tension in steel. (14000#□")
E_c= modulus of elasticity of concrete in compression. (3,000,000#)
E_s= " " " " steel. (30,000,000#)

$r = \frac{E_s}{E_c}$=ratio of moduli of elasticity of steel to concrete. (10)
d = distance from outside compressive fibre to center of (gravity of steel)
xd= " " " " surface to neutral axis in a beam having steel at a depth(d)below the outside compressive surface.
x = ratio of depth of neutral axis to depth (d) of steel.
M_r= moment of resistance.
M_b= bending moment.

All dimensions are in inches(except length or span which is in feet) ,stress in pounds per square inch, and moments in inch pounds.

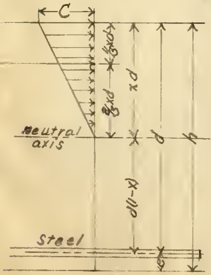


Figure 5.

"Referring to figure #5 there is found by the internal resisting forces a couple. One arm represents the total compression in the concrete acting through its center of compression,viz.,the center of gravity of the triangular area representing the compression and having for its base C and its height xd. The other arm represents the total tension in the steel acting through the center of gravity of the rod or group of rods.

For equilibrium in the beam,these two forces,parallel to each other and acting in opposite directions,must be equal. In other words,the total compression in the concrete must equal the total tension in the steel.

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$$\frac{\text{Deformation in steel}}{\text{Deformation in outside compressive concrete fibres}} = \frac{d(1-x)}{xd}$$

Therefore $\frac{S/E_s}{C/E_c} = \frac{d(1-x)}{xd}$ since deformation $= \frac{\text{Stress per sq.inch.}}{\text{Modulus of elasticity.}}$

Solving the above for $x = \frac{1}{1 + \frac{S}{Cr}}$ or $C = \frac{S\,x}{r(1-x)}$

The total tension in the steel must equal the total compression in the concrete, therefore by multiplying(S) by the area of the steel (pbd), and equating it to 1/2 C(xd) times (b) the breadth of the beam and solving for(pS) we have $\left(pS = \frac{Cx}{2}\right)$ from which we see by substituting the above value of (x) gives for $p = \frac{1}{\frac{2S}{C}\left(1 + \frac{S}{Cr}\right)}$

For any given percentage of steel the values of S and C cannot be assumed independently, as they bear a constant ratio to each other.

By substituting the value of C found previously for C in the last formula we have $p = \frac{x}{2}\,\frac{x}{(1-x)r}$ solving this quadratic equation and adopting the positive sign before the square root

$$x = -rp + \sqrt{2rp + (rp)^2}$$

The moment of resistance is obtained from the couple by taking moments about the center of compression in the concrete, using for the force the total tension in the steel, which, as above, is(pSbd) times the arm $\left(d - \frac{xd}{3}\right)$ therefore:-

$M_r = pSbd^2\left(1 - \frac{x}{3}\right)$ or the moment of resistance may also be expressed in terms of compression in the concrete by combining the equations $pS=Cx/2$ and the last one derived, giving:-

$$M_r = \frac{Cxbd^2}{2}\left(1 - \frac{x}{3}\right)$$

Design of Sections.

The design of all sections is the same except that for all points but the crown, the horizontal thrust must be multiplied by the secant of the angle of slope at that point; therefore the calculation for the crown section only will be given in detail; the data for the other sections is given in the tables.

The maximum vertical negative and positive live load bending moments for the design of the section were found by adding the negative bending moments at the section due to the loads at the various panel points for the maximum negative bending moment, and the positive for the maximum positive bending moment. In the case of the dead load vertical bending moment and also the horizontal dead load bending moment an algebraic summation of the bending moments due to the loads at the various panel points was made and these were used in figuring the section. For the horizontal live load bending moment the position or the loads giving maximum vertical live load bending moment were noted and the horizontal live load bending moments caused at the same points were algebraicly added for the horizontal live load bending moment to be used in figuring the section. This was done because this bending moment would be the horizontal live load bending moment acting at the section when the greatest vertical live load bending moment was acting.

All live load stresses were doubled to reduce them to equivalent dead load stresses.

By the above rules the bending moments at the crown section were as follows:-

	V-L.L.	V-D.L.	H-L.L.	H-D.L.	Total-max.
Max- negative "B.M."	-482,000	-305,700	-60,200	-108,800	-1,498,920
" positive "	719,300	-305,700	(-4,200)	108,800	-1,015,720

As shown by the above table the maximum negative bending moment is the greater; therefore the section was calculated for this bending moment.

The summation of the thrusts due to the live loads and the dead load = 126,971#. As stated above the temperature bending moment formula was $M = 387\ I/k$ in which $I = Mx'/f_c$.

Since part of the strength of the section will be used in resisting the bending moment and part in resisting thrust, it is necessary before designing the section for bending moment to find the amount of the strength that is left to resist the bending moment after the thrust has been taken care of; this value of f_c in the formula $I = Mx'/f_c$ is found as follows:- A depth of 37 inches was assumed for the crown section and this multiplied by 12 gives 444 square inches for a section of the arch one foot wide; dividing the total thrust 126,971# by 444 gives 286# the amount necessary to take care of the thrust.

Subtract from the safe allowable stress for concrete in pounds per square inch (which in this case was taken as 1100# or 1/2 of the ultimate strength, thus giving a factor of safety of 2 for dead load and 4 for live load, the live load stresses having already been multiplied by 2 to reduce them to equivalent dead load) the 286# or 1100# - 286# = 814# per square inch available to take care of the bending moment. This is the f_c. Now by formula $x = \frac{1}{1 + S/Cr}$ we find that the neutral axis is at a point .368 of the depth of the beam from the compression surface.

$$x = \frac{1}{1 + \frac{14,000}{814 \cdot 10}} = .368$$

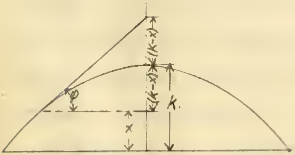
Substituting this value of x in the formula $I = Mx'/f_c$ where $x' = xd$, we find $I = \frac{1,498,920 \times .368 \times 37}{814} = 25073$

$M_t = 387\ I/k = 387 \times 25073/18 = 539069$ inch pounds.

$T_t = 96\ I/k^2 = 96 \times 25073/324 = 7428\#$

Adding 539069 to the 1,498,920 inch pounds bending moment due to the loads, we have for the total bending moment 2,037,989 inch pounds; and adding 7428# to 126,971# the thrust due to the loads we have 134,399# for the total thrust. Solving again as before for a new value of f_c using the above values of total thrust and bending moment, we find $f_c = 797\#$, solving again for x with 797# as the value of C we get x = .362. Now substituting in formula $M_r = \frac{Cxbd^2}{2}(1-x/3)$, we get d = 36.6 or 37 inches which checks with the value assumed.

Figure #6.



The method of obtaining the thrust was the only point in which the figuring of the other sections differed from the crown section. In all other sections the horizontal thrust had to be multiplied by the secant of the angle of slope of the curve at that point, in order to get the direct thrust on the section. Owing to the fact that the tangent to a parabola cuts off a distance above the crown on a perpendicular at the crown equal to the distance that the point is below the crown, the angle of slope at the various points was obtained by the formula:- Tangent $\varphi = \frac{2(k - x)}{y}$, the derivation of which can be seen in figure #6

Steel.

By formula $pS = Cx/2$ we find that p or the percentage of the section that must be steel equals (.0104) or for a

section 37 inches by 12 inches the area of the steel necessary to take the tension equals 4.75 square inches. In this manner the steel was calculated for each section; since the maximum bending moment was positive in some cases and negative in others, the maximum amount of steel was in some cases at the introdos and in others at the extrados. The steel was figured for the introdos at the various sections and the maximum of these values was run entirely across the arch as introdos reinforcement. In like manner the maximum amount of steel figured for the various sections of the extrados, except the abutment section, was run entirely across the arch as extrados reinforcement. At the abutment section extra steel was put in to take the unusually large bending moment. 3/4" square rods were placed 18" C.to C. on both extrados and introdos for transverse reinforcing. 1/2" square rods were placed as shear reinforcing at an angle of 45 degrees to the line of stress in such a manner that no plane perpendicular to the line of stress was without reinforcement. This steel was added as a precaution only, because the sections as figured for bending moment were larger than was necessary to take care of the shear. Two inches of concrete was placed outside of all reinforcement. All steel was spliced by being lapped 3 feet and bent over 1 foot, perpendicular to the line of stress.

In order to facillitate the erection of forms the extrados and introdos were struck as arcs of circles as shown on the drawing, in such a manner as to give the required section at the panel points.

Abutments.

The dimensions of the abutment were made such that the resultant of the thrust of the arch and the weight of the abutment and arch fell within the middle third of the base. This was obtained by a graphical method not shown. The foundation consisted of piles placed 3 feet center to center in both directions,and extending 2 feet into the concrete abutment. The abutment was reinforced at the bottom by 2 inch square rods spaced 3 feet center to center in both directions,one set being placed 4 inches above the other. The steel of the extrados reinforcing was extended along the entire upper surface of the abutment.

Retaining Walls.

In figuring the retaining wall the live load was taken as equivalent to 10 feet of additional earth filling. The section of the retaining wall was figured for points 12, 24,and 40 feet from the top,by the formula for beam cross section previously given. The moment at the section was obtained under the usual assumption that the center of pressure was at a point 2/3 of the distance from the top of the wall to the section. The retaining wall being considered as a cantilever,the vertical reinforcing was run down into the arch ring and anchored in the manner explained above. Horizontal reinforcing was placed at the outer surface of the wall to take any tension that might come upon the wall due to its tendency to buldge in a horizontal plane. The vertical reinforcing was 1-1/4 inch square bars for the entire height,with extra 1-1/4 inch bars on the lower 10 feet,all spaced 4 inches

center to center. The horizontal reinforcing was of 3/4 inch square bars spaced 18 inches center to center.

As stated several times previously certain formulas and tables were taken from Prof. Greene's book" Trusses and Arches Part 3 ",and from Mr.Colpitts pamphlet "The Calculation of the Stresses and the Practical Design of Structures of Steel Concrete". Mr Colpitts formulas and tables are in most cases convenient modifications of those given by Prof. Greene. Since Prof.Greene has carefully derived in considerable detail all of the formulas and tables in his book the reader is referred to it if a more detailed discussion of any formula or table is desired.

O. T. Allen
C. S. Holcomb

TABLE-----# /

Horizontal thrusts due to vertical live loads.

Panel	1	2	3	4	5	Total
	1030#	3862#	7664#	11106#	12494#	36156#

- -

Horizontal thrusts due to vertical dead loads.

Panel	1	2	3	4	5	Total
	2776#	6628#	8583#	8372#	7840#	34199#

- -

Horizontal thrusts due to horizontal live loads.

Equals sum of horizontal live loads = 5410#

- -

Horizontal thrusts due to horizontal dead loads.

Equals sum of horizontal dead loads = 9640#

- -

Horizontal thrusts due to temperature = 10217#

- -

Thrusts used in the design of sections ie. twice the live load thrusts,plus the dead load thrusts,plus the temperature thrust , added together and multiplied by the secant of the angle of , slope at the section. Thrust = 137,188# x secant φ.

Panel	0	1	2	3	4	5
	184,655#	169,015#	155,845#	145,830#	139,383#	137,188#

- -

Ordinates to the parabola calculated from $y=k(1 - x^2/c^2)$, k=18'
c=40'

Panel point	0	1	2	3	4	5
Ordinate	0.00'	6.48'	11.52'	15.12'	17.28'	18.00'

TABLE-------# 2

Vertical live and dead and horizontal live and dead loads.

Panels	1	2	3	4	5
V--L.L.	7610#	9060#	10430#	11580#	12000#
V--D.L.	20500#	15550#	11680#	8730#	7530#
H--L.L.	1830#	1640#	1250#	690#	0000
H--D.L.	4920#	2800#	1400#	520#	0000

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Bending Moments
for
Vertical Live Loads

(3)

Inch Pounds

Load on \ Moment at	0	1	2	3	4	5
9	+ 80400	+ 21900	− 18300	− 43800	− 47500	− 36600
8	+ 278400	+ 69600	− 73900	− 152200	− 160900	− 104400
7	+ 475300	+ 95100	− 155100	− 270000	− 250200	− 100000
6	+ 533400	+ 61000	− 222300	− 311100	− 255000	+ 88900
5	+ 357100	− 34500	− 213000	− 178500	+ 69100	+ 541500
4	000000	− 144400	− 94500	+ 144400	+ 577800	+ 88900
3	− 365200	− 180100	+ 140100	+ 595500	+ 180100	− 100000
2	− 556800	− 78300	+ 465400	+ 208800	+ 17400	− 104400
1	− 442200	+ 186300	+ 102300	+ 40200	− 73000	− 36600
Max. −	− 1364200	− 437300	− 777100	− 955600	− 786600	− 482000
" +	+ 1724600	+ 433900	+ 707800	+ 988900	+ 844400	+ 719300

Bending Moments
for
Horizontal Live Loads

(4)

Load on \ Moment at	0	1	2	3	4	5
9	+ 15700	+ 4300	− 3600	− 8200	− 9500	− 7200
8	+ 34200	+ 8500	− 9100	− 18600	− 20000	− 13400
7	+ 35300	+ 7600	− 10600	− 19500	− 19100	− 9500
6	+ 19500	+ 3500	− 6700	− 11000	− 9400	− 2100
5	— — —	— — —	— — —	— — —	— — —	— — —
4	− 20500	− 3600	+ 7400	+ 12200	+ 11100	− 2100
3	− 49700	− 8100	+ 21100	+ 38000	+ 9700	− 9500
2	− 83400	− 3500	+ 56300	+ 25000	+ 1800	− 13400
1	− 89000	+ 35500	+ 19700	+ 7200	− 1600	− 7200
Max. −	242600	− 15200	− 30000	− 57300	− 59600	− 64400
+	104700	+ 59400	+ 104500	+ 82400	+ 22600	+ —

Bending Moments
for
Vertical Dead Loads

(5)

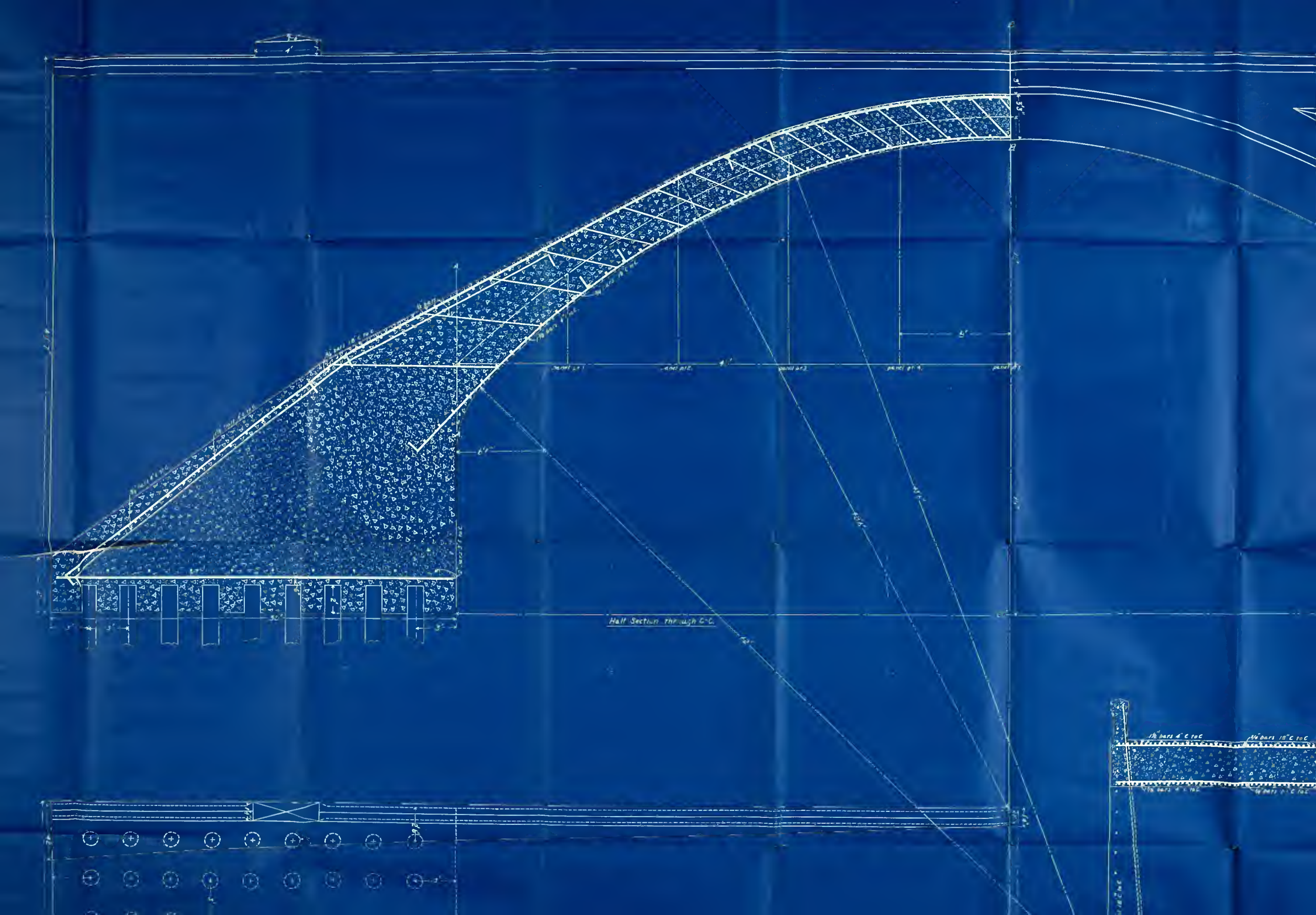
Inch Pounds

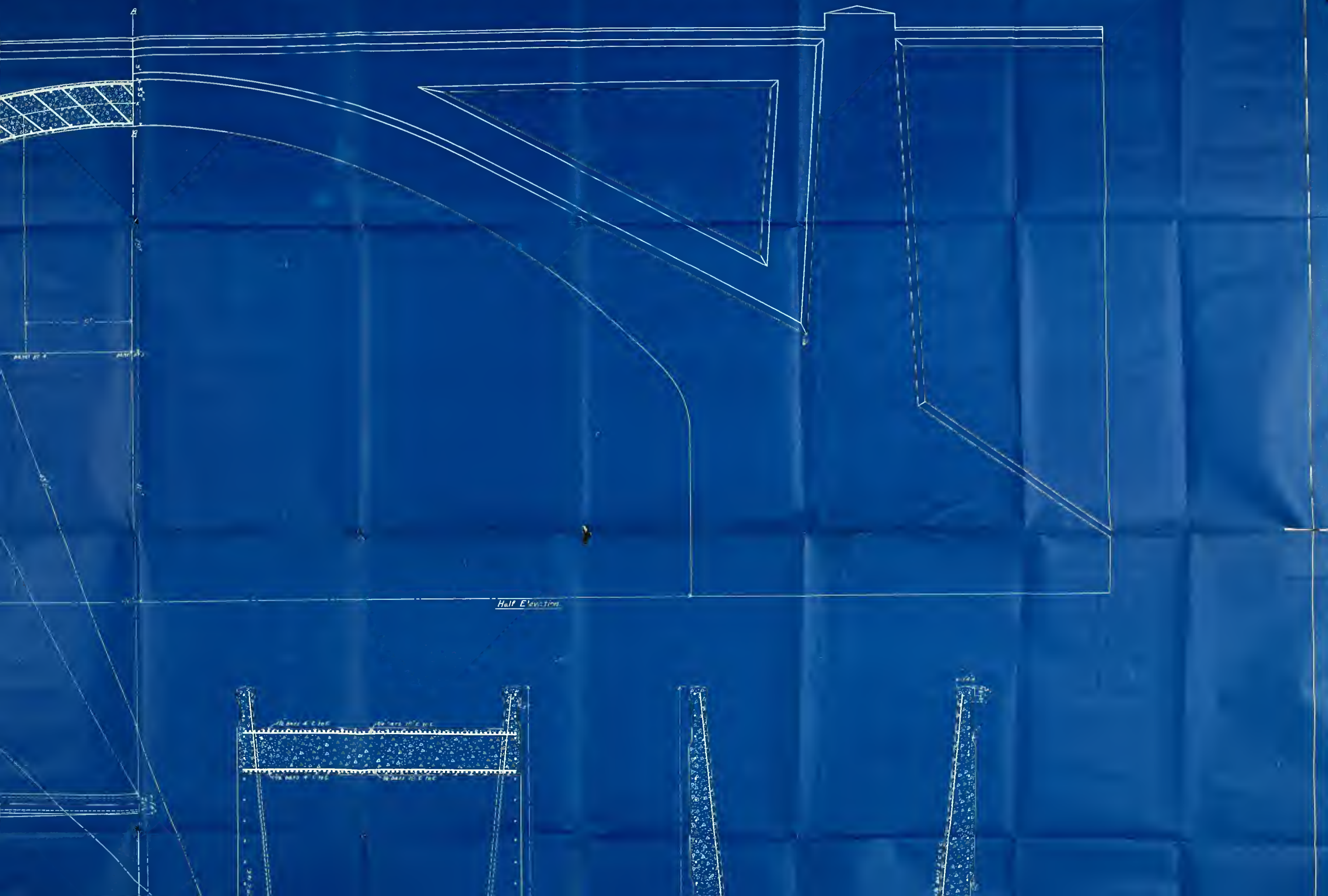
Load on \ Moment at	0	1	2	3	4	5
9	+216500	+59000	-49200	-118100	-127900	-98400
8	+477700	+119400	-126900	-261200	-276200	-179100
7	+532300	+106500	-173700	-302600	-280200	-112100
6	+402000	+46100	-167500	-234500	-155000	+67000
5	+223900	-21700	-133600	-112000	+43300	+339500
4	000	-108900	-71200	+108900	+435600	+67000
3	-409100	-201700	+156900	+666900	+201700	-112100
2	-955400	-134400	+798600	+358300	+29900	-179100
1	-1190600	+501800	+275500	+108200	-19700	-98400
Max. –	-2555100	-466700	-722100	-1028400	-859000	-779200
" +	+1852400	+832800	+1231000	+1242300	+710500	+473500

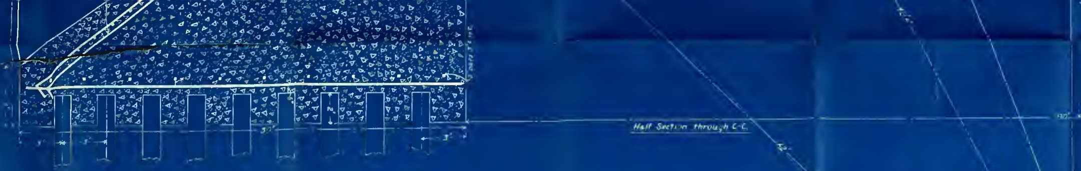
Bending Moments
for
Horizontal Dead Loads

(6)

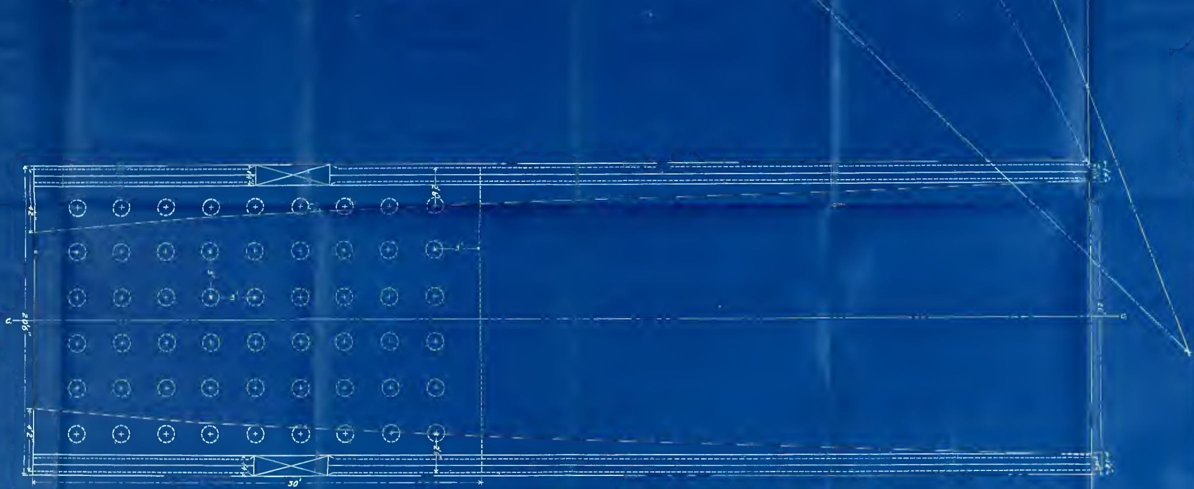
Load on \ Moment at	0	1	2	3	4	5
9	+42500	+11500	-9700	-22100	-25700	-19500
8	+58500	+14600	-15600	-31700	-34300	-22700
7	+34600	+8600	-11800	-21900	-21400	-10600
6	+14800	+2600	-5100	-8300	-7100	-1600
5						
4	-15500	-2700	+5600	+9300	+8400	-1600
3	-55700	-9100	+23700	+42600	+10800	-10600
2	-142600	-6100	+96300	+42800	+3000	-22700
1	-237700	+95600	+53100	+19500	-4400	-19500
Max. –	-451500	-17900	-42200	-84000	-92900	-108800
+	+150400	132900	+178700	+114300	+22200	


Half Section through C-C
panel pt. 1
panel pt. 2
panel pt. 3
panel pt. 4
panel pt. 5
5'
6"
1½" bars 4" C to C
¾" bars 18" C to C
A
B
C

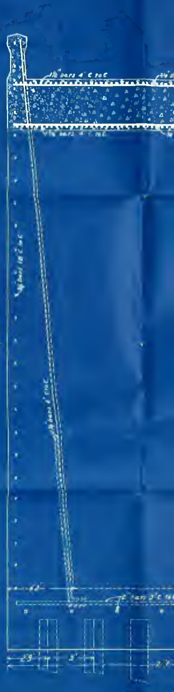

B
B
Half Elevation



Half Section through C-C.



Half Plan.



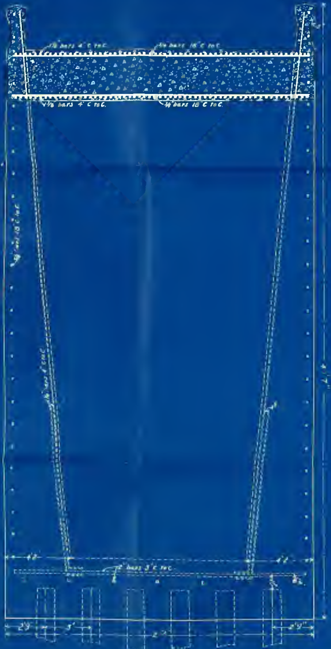
Section th

Note.

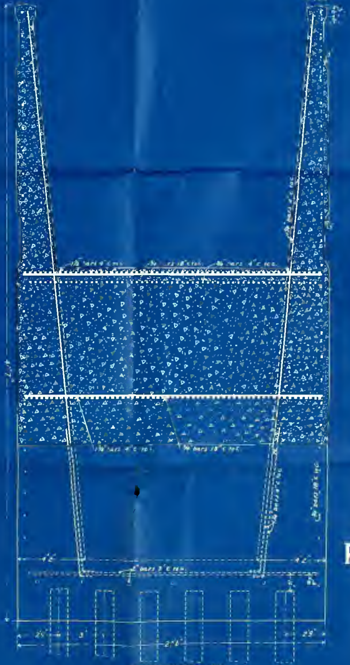
All bars square and of medium steel.
All bars lapped 3' at splices.
All bars anchored by bending 1' at both ends 90° with the axis of the bar.

Data.

Line of stress | Parabolic. | 80' span. | 18' rise.
Span of bridge 80'.
Clear-way of bridge 34'4½".
Extreme width 26'6".
Extreme height 40'7".
Extreme length 140'.



Section through B-B



Section through A-A

DESIGN OF
REINFORCED CONCRETE
RAILROAD
ARCH BRIDGE

ARMOUR INST. TECHNOLOGY
CHICAGO ILL. MARCH '06

THESIS
C. S. Holcomb